

Screened Casimir forces

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We demonstrate that a very recently obtained formula for the force on a slab in a material planar cavity based on the calculation of the vacuum Lorentz force [C. Raabe and D.-G. Welsch, Phys. Rev. A, **71**, 013814 (2005)] describes a (medium) screened Casimir force and, in addition to it, a medium-assisted force. The latter force also describes the force on the cavity medium. For dilute media, it implies the atom-mirror interaction of the Casimir-Polder type at large and of the Coulomb type at small atom-mirror distances of which the sign is insensitive to the polarizability type (electric or magnetic) of the atom.

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It is well known that an atom in the vicinity of a body (mirror) experiences the Casimir-Polder force [1] and, at smaller distances, its nonretarded counterpart, the van der Waals force. Consequently, being a collection of atoms, every piece of a medium in front of a mirror should experience the corresponding force. Despite this, a number of approaches to the Casimir effect [2] in material systems lead to the result that the Casimir force on the medium between two mirrors vanishes and that the only existing force is that between the mirrors [3, 4, 5] (see also text books [6, 7] and references therein). To overcome this "unphysical" result, usually derived by calculating the Minkowski stress tensor [3, 5] but also obtained using other methods [4, 7], Raabe and Welsch very recently [8] suggested a Lorentz-force approach to the Casimir effect. In their approach the force on a body is obtained by calculating the sum of the vacuum Lorentz forces acting on its constituents. Evidently, this method should lead to a nonzero force on the medium between the mirrors. As an application of their approach, Raabe and Welsch derived a formula for the force on a magnetodielectric slab in a magnetodielectric planar cavity, as depicted in Fig. 1. In this paper we i) demonstrate that, according to the Raabe and Welsch formula, the total force on the slab actually consists of a medium-screened Casimir force and a medium-assisted force and ii) point out a few unexpected results coming from the unusual properties of the latter force.

In the Lorentz-force approach, the force on the slab in the configuration of Fig. 1 is given by [8]

$$f(d_1, d_2) = -\frac{\hbar}{8\pi^2} \int_0^\infty d\xi \int_0^\infty \frac{dk k}{\kappa} \mu \times \sum_{q=p,s} [g_{q2}(i\xi, k; 0) - g_{q1}(i\xi, k; d_1)], \quad (1)$$

where

$$\kappa(\xi, k) = \sqrt{n^2(i\xi) \frac{\xi^2}{c^2} + k^2} \quad (2)$$

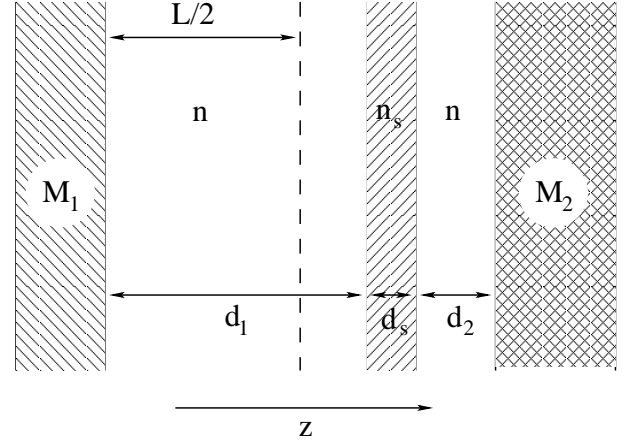


FIG. 1: A slab in a planar cavity shown schematically. The (complex) refractive index of the slab is $n_s(\omega) = \sqrt{\epsilon_s(\omega)\mu_s(\omega)}$ and that of the cavity $n(\omega) = \sqrt{\epsilon(\omega)\mu(\omega)}$. The cavity walls are described by their reflection coefficients $r_1^q(\omega, k)$ and $r_2^q(\omega, k)$, with k being the in-plane wave vector of a wave. The arrow indicates the direction of the force on the slab.

is the perpendicular wave vector in the cavity at the imaginary frequency, and [9]

$$g_{q2}(i\xi, k; 0) - g_{q1}(i\xi, k; d_1) = - \left\{ 4\kappa^2 \left(\delta_{qs} + \frac{1}{n^2} \delta_{qp} \right) r^q + \frac{\xi^2}{c^2} (n^2 - 1) [(1 + r^q)^2 - t^{q2}] \Delta_q \right\} \frac{r_2^q e^{-2\kappa d_2} - r_1^q e^{-2\kappa d_1}}{N^q}, \quad (3)$$

$$N^q = 1 - r^q (r_1^q e^{-2\kappa d_1} + r_2^q e^{-2\kappa d_2}) + (r^{q2} - t^{q2}) r_1^q r_2^q e^{-2\kappa(d_1+d_2)}, \quad (4)$$

with $\Delta_q = \delta_{qp} - \delta_{qs}$. Here $r^q = r_{1/2}^q = r_{2/1}^q$ and $t^q = t_{1/2}^q = t_{2/1}^q$ are Fresnel coefficients for the (whole) slab

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given by

$$r^q(i\xi, k) = \rho^q \frac{1 - e^{-2\kappa_s d_s}}{1 - \rho^{q2} e^{-2\kappa_s d_s}}, \quad t^q(i\xi, k) = \frac{(1 - \rho^{q2})e^{-\kappa_s d_s}}{1 - \rho^{q2} e^{-2\kappa_s d_s}}, \quad (5)$$

where

$$\rho^p(i\xi, k) = \frac{\varepsilon_s \kappa - \varepsilon \kappa_s}{\varepsilon_s \kappa + \varepsilon \kappa_s}, \quad \rho^s(i\xi, k) = \frac{\mu_s \kappa - \mu \kappa_s}{\mu_s \kappa + \mu \kappa_s}, \quad (6)$$

are the single-interface medium-slab ($\rho^q = r_{1s}^q = r_{2s}^q$) Fresnel reflection coefficients.

1. Medium-screened Casimir force and medium-assisted force

Combining Eqs. (1) and (3), we see that f naturally splits into two rather different components

$$f(d_1, d_2) = f^{(1)}(d_1, d_2) + f^{(2)}(d_1, d_2), \quad (7)$$

where

$$f^{(1)}(d_1, d_2) = \frac{\hbar}{2\pi^2} \int_0^\infty d\xi \int_0^\infty dk k \kappa \times \sum_{q=p,s} \left(\mu \delta_{qs} + \frac{1}{\varepsilon} \delta_{qp} \right) r^q \frac{r_2^q e^{-2\kappa d_2} - r_1^q e^{-2\kappa d_1}}{N^q}, \quad (8)$$

and

$$f^{(2)}(d_1, d_2) = \frac{\hbar}{8\pi^2 c^2} \int_0^\infty d\xi \xi^2 \mu (n^2 - 1) \int_0^\infty \frac{dk k}{\kappa} \times \sum_{q=p,s} [(1 + r^q)^2 - t^{q2}] \Delta_q \frac{r_2^q e^{-2\kappa d_2} - r_1^q e^{-2\kappa d_1}}{N^q}. \quad (9)$$

Equation (8) differs in two respects from the formula for the Casimir force in a dielectric cavity obtained through the Minkowski tensor calculation [5]. First, the Fresnel coefficients refer here to a magnetodielectric system. Another new feature in Eq. (8) is the (effective) screening of the force through the multiplication of the contributions coming from TE- and TM-polarized waves by μ and $1/\varepsilon$, respectively. This provides a simple recipe how to adapt the traditionally obtained formulas for the Casimir force to the present approach.

Clearly, $f^{(2)}$ owes its appearance to the cavity medium, note that it vanishes when $n = 1$, and is therefore a genuine consequence of the Lorentz-force approach. Another unique feature of $f^{(2)}$ is the dependence on the properties of the cavity mirrors coming from its proportionality to $\Delta_q r_i^q$ rather than to $r^q r_i^q$, as is the case with $f^{(1)}$. Owing to this property the sign of each term in Eq. (9) depends on whether the corresponding mirror is dominantly conductive (dielectric) or permeable irrespective of the properties of the slab. We illustrate this by calculating the

force on an ideally reflecting slab in a semi-infinite, e.g., $d_1 \rightarrow \infty$, cavity with an ideally reflecting mirror. Letting $t^q = 0$, $r^q = \pm \Delta_q$ [the minus sign is for an infinitely permeable slab, see Eqs. (5) and (6)], $r_2^q = \pm \Delta_q$, and assuming $d_2 \equiv d$ so large that ε and μ can be replaced by their static values ε_0 and μ_0 , respectively, the integrals in Eqs. (8) and (9) become elementary and we find

$$f_{\text{id}}^{(1)}(d) = \left\{ \begin{array}{c} 1 \\ -7/8 \end{array} \right\} \frac{\hbar c \pi^2}{15 \cdot 2^5 d^4} \sqrt{\frac{\mu_0}{\varepsilon_0}} \left(1 + \frac{1}{n_0^2} \right), \quad (10)$$

$$f_{\text{id}}^{(2)}(d) = \pm \left\{ \begin{array}{c} 1 \\ 7/8 \end{array} \right\} \frac{\hbar c \pi^2}{45 \cdot 2^5 d^4} \sqrt{\frac{\mu_0}{\varepsilon_0}} \left(1 - \frac{1}{n_0^2} \right). \quad (11)$$

Here, the first (second) line in the curly brackets corresponds to a system with the slab and the mirror of the same (different [10]) type (conductive or permeable), whereas the sign of $f_{\text{id}}^{(2)}$ depends on whether the mirror is conductive (+) or permeable (-). The above equations therefore describe the force on the slab in four possible different configurations. Thus, the result quoted by Raabe and Welsch [8] (the second line is for optically dense cavity media)

$$f_{\text{id}}^{\text{cc}}(d) = \frac{\hbar c \pi^2}{720 d^4} \sqrt{\frac{\mu_0}{\varepsilon_0}} \left(2 + \frac{1}{n_0^2} \right) \simeq \frac{\hbar c \pi^2}{360 d^4} \sqrt{\frac{\mu_0}{\varepsilon_0}}, \quad (12)$$

is recovered when the mirror and the slab are both conductive (cc). Note that in this case and for dense media, $f_{\text{id}}^{(2)}$ is only three times smaller than $f_{\text{id}}^{(1)}$. We also observe that, when the mirror and the slab are both permeable (pp), the above equations imply the total force $f_{\text{id}}^{\text{pp}} = [(n_0^2 + 2)/(2n_0^2 + 1)] f_{\text{id}}^{\text{cc}} (\simeq 1/2 f_{\text{id}}^{\text{cc}}$ for dense media). When the mirror and the slab are of different type (cp or pc), however, we find $f_{\text{id}}^{\text{cp}} = -(7/8)[(n_0^2 + 2)/(2n_0^2 + 1)] f_{\text{id}}^{\text{cc}}$ whereas $f_{\text{id}}^{\text{pc}} = -(7/8) f_{\text{id}}^{\text{cc}}$. Finally, we note that the force on the slab in a finite cavity is given by $f_{\text{id}}(d_1, d_2) = f_{\text{id}}(d_2) - f_{\text{id}}(d_1)$ and is therefore obtained by combining the above results independently for each part of the cavity.

2. Force on the cavity medium and on an atom

Specially, in the case $n_s = n$, $f = f^{(2)}$ describes the force on a layer of the medium in the cavity. Letting $n_s = n$ [$\rho^q = 0$ in Eq. (5)], we have $r^q = 0$ and $t^q = e^{i\beta d_s}$ in Eq. (9), so that we may write

$$f^{(2)}(d_1, d_2) = \int_{d_2}^{d_2+d_s} f_2(z) dz - \int_{d_1}^{d_1+d_s} f_1(z) dz, \quad (13)$$

where the force densities $f_i(z)$ are given by

$$f_i(z) = \frac{\hbar}{4\pi^2 c^2} \int_0^\infty d\xi \xi^2 \mu (n^2 - 1) \int_0^\infty dk k e^{-2\kappa z} \times \sum_{q=p,s} \Delta_q \frac{r_i^q}{1 - r_1^q r_2^q e^{-2\kappa L}}, \quad (14)$$

with $L = d_1 + d_2 + d_s$ being the cavity length. For a semi-infinite cavity (obtained by letting either $d_1 \rightarrow \infty$ or $d_2 \rightarrow \infty$), $f_i(z)$ become characteristic functions only of the medium and the corresponding mirror, so that we may drop the index i denoting the mirror. As follows from the above definition, positive $f(z)$ means attraction between the medium and the mirror. Clearly, for a dilute medium, the force density is related to the force on an atom $f_{\text{at}}(z)$ through

$$f(z) = N f_{\text{at}}(z), \quad (15)$$

where N is the atomic number density. The behavior of $f(z)$ is therefore the same as that of $f_{\text{at}}(z)$ discussed below.

Assuming the medium dilute and letting

$$n^2(i\xi) - 1 \simeq 4\pi N\alpha(i\xi), \quad \alpha(i\xi) = \alpha_e(i\xi) + \alpha_m(i\xi), \quad (16)$$

where $\alpha_{e(m)}$ is the electric (magnetic) polarizability of the atom, from Eq. (14) (with $L \rightarrow \infty$) we find

$$f_{\text{at}}(z) = \frac{\hbar}{\pi c^2} \int_0^\infty d\xi \xi^2 \mu \alpha \int_0^\infty dk k e^{-2\kappa z} \times [r^p(i\xi, k) - r^s(i\xi, k)]. \quad (17)$$

The integral over ξ here effectively extends up to a frequency Ω beyond which the mirror becomes transparent. Therefore, at small atom-mirror distances $\Omega z/c \ll 1$, the main contribution to the integral comes from large k 's ($k \sim 1/z$). In this k -region, we may approximate the integrand with its nonretarded (nr) counterpart obtained formally by letting $\kappa = k$ and $\kappa_l = k$ for all layers of the mirror. In this way, and by making the substitution $u = 2\kappa z$, we obtain

$$f_{\text{at}}(z) = \frac{\hbar}{4\pi c^2 z^2} \int_0^\infty d\xi \xi^2 \mu \alpha \int_0^\infty du u e^{-u} \times \left[r_{\text{nr}}^p(i\xi, \frac{u}{2z}) - r_{\text{nr}}^s(i\xi, \frac{u}{2z}) \right]. \quad (18)$$

For a single-medium mirror, $r_{\text{nr}}^q(i\xi, u/2z)$ are independent of u [see Eq. (6), with $k \rightarrow \infty$ and $\{\varepsilon_s, \mu_s\} \rightarrow \{\varepsilon_m, \mu_m\}$] and for this classical configuration we find

$$f_{\text{at}}(z) = \frac{\hbar}{4\pi c^2 z^2} \int_0^\infty d\xi \xi^2 \mu \alpha \left[\frac{\varepsilon_m - \varepsilon}{\varepsilon_m + \varepsilon} - \frac{\mu_m - \mu}{\mu_m + \mu} \right] \quad (19)$$

rather than the common van der Waals force.

To find the large- z behavior of $f_{\text{at}}(z)$, we make the standard substitution $\kappa = n\xi p/c$ in Eq. (17). This gives

$$f_{\text{at}}(z) = \frac{\hbar}{\pi c^4} \int_0^\infty d\xi \xi^4 \mu n^2 \alpha \int_1^\infty dp p e^{-2n\xi p z/c} \times [r^p(i\xi, p) - r^s(i\xi, p)], \quad (20)$$

where $r^q(i\xi, p)$ are obtained from $r^q(i\xi, k)$ by letting $\kappa_l \rightarrow n(\xi/c)s_l$, with $s_l = \sqrt{p^2 - 1 + n_l^2/n^2}$ for all relevant layers. Thus, for example, for a single-medium mirror with the refraction index n_m we have

$$r^p(i\xi, p) = \frac{\varepsilon_m p - \varepsilon s_m}{\varepsilon_m p + \varepsilon s_m}, \quad r^s(i\xi, p) = \frac{\mu_m p - \mu s_m}{\mu_m p + \mu s_m}. \quad (21)$$

Now, for large z , the contributions from the region $\xi \simeq 0$ dominate the integral in Eq. (20) and we may approximate the frequency-dependent quantities with their static values (denoted by the subscript 0). In this case, the integral over ξ becomes elementary and we find

$$f_{\text{at}}(z) = \frac{3\hbar c \alpha_0}{4\pi n_0 \varepsilon_0 z^5} \int_1^\infty \frac{dp}{p^4} [r^p(0, p) - r^s(0, p)]. \quad (22)$$

For a perfectly conductive mirror, the value of the above integral is $2/3$. As seen, since $n_0 \varepsilon_0 \simeq 1$ for dilute media, in this case $f_{\text{at}}(z)$ at large distances is effectively three times smaller than the Casimir-Polder force [1]. However, contrary to the behavior of the Casimir-Polder force [11], for an atom near a dominantly conductive (permeable) mirror Eq. (22) [as well as Eqs. (17)-(19)] predicts an attractive (repulsive) force irrespective of the polarizability of the atom [12].

We end this short discussion by noting that ten years ago Zhou and Spruch (ZS) considered the atom-mirror interaction for an atom in a dielectric cavity [4]. Their result for the force on an atom in a semi-infinite cavity is given by Eq. (17) when letting

$$r^p(i\xi, k) \rightarrow \left(2 \frac{\kappa^2 c^2}{n^2 \xi^2} - 1 \right) r^p(i\xi, k) \quad (23)$$

in the last factor of the integrand. Proceeding as before, we see that for the leading term at small distances, this formula gives the van der Waals force

$$f_{\text{at}}^{\text{ZS}}(z) = \frac{\hbar}{8\pi z^4} \int_0^\infty d\xi \xi \frac{\alpha}{\varepsilon} \int_0^\infty du u^3 e^{-u} r_{\text{nr}}^p(i\xi, \frac{u}{2z}). \quad (24)$$

At large distances, it leads to

$$f_{\text{at}}^{\text{ZS}}(z) = \frac{3\hbar c \alpha_0}{4\pi n_0 \varepsilon_0 z^5} \int_1^\infty \frac{dp}{p^4} [(2p^2 - 1)r^p(0, p) - r^s(0, p)], \quad (25)$$

thus reproducing the Casimir-Polder result [1] in the case of a perfectly conductive mirror and an empty cavity. Of course, one must keep in mind that the physical situation considered by Zhou and Spruch is substantially different from that considered in this work. They calculated the force on an atom embedded in the medium and not the force on an atom of the medium, as we have done. Thus, although we may formally let $n_0 \varepsilon_0 \simeq 1$ in Eq. (22), as appropriate for a dilute medium, we cannot interpret this as effectively the force on the atom in vacuum. In other words, contrary to Zhou and Spruch, we cannot, in principle, reproduce the Casimir-Polder result starting from the medium-assisted force and, for this reason, we may regard Eq. (22) as describing a (medium) screened Casimir-Polder force.

In conclusion, the Raabe and Welsch result for the force on a slab in a planar cavity naturally splits into a formula for a medium-screened Casimir force and into a formula for a medium-assisted force. The latter force is in an unusual way related to the properties of the cavity medium and mirrors. It also describes the force on the

cavity medium and, for dilute media, implies the atom-mirror interaction of the screened Casimir-Polder type at large and of the Coulomb type at small atom-mirror distances. Contrary to the sign of the Casimir-Polder interaction, the sign of the medium-assisted interaction is insensitive to the polarizability type of the atom. Ev-

idently, to understand these results, a microscopic consideration of the atom-mirror interaction for an atom of the medium in the vicinity of a mirror is needed.

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